

CERN-TH/98-92
hep-ph/9803425

UPGRADING OF PRECISION CALCULATIONS FOR ELECTROWEAK OBSERVABLES

Dmitri Bardin¹

and

Giampiero Passarino²

¹ Laboratory of Nuclear Problems, JINR, Dubna, Russia*

² Dipartimento di Fisica Teorica, Università di Torino and
INFN, Sezione di Torino, Turin, Italy

ABSTRACT

A critical assessment is given of the comparison between the new versions of the programs TOPAZ0 40i and ZFITTER 510. The relevance for precision calculations around the Z resonance is briefly discussed.

*) Present address: TH Division, CERN, 1211 Geneva 23, Switzerland.
e-mail address: Dmitri Bardine@cern.ch.

1 Introduction

In 1995 the Z phase of LEP came to an end and at present the ultimate analysis of the data is imminent. This involves in particular the completion of the line-shape analysis, including the final LEP energy calibration. Consequently, the safest possible estimate of the theoretical accuracy is of the utmost importance. It should be noted that the LEP 1 data (1990–1995) were taken in the energy (\sqrt{s}) range $|\sqrt{s} - M_Z| < 3$ GeV and consist of the hadronic and leptonic cross sections, the leptonic forward–backward asymmetries, the various polarization asymmetries, the partial widths, and the quark forward–backward asymmetries. All this makes it mandatory to assess the theoretical precision of the available programs for different channels and for different pseudo-observables.

In this note we focus on the calculation of the pseudo-observables. Independently of the renormalization procedure that is used, the matrix element for $Z \rightarrow f\bar{f}$ will be written as

$$\mathcal{M}_{f\bar{f}}^Z = \bar{u}_f \phi_Z \left(\mathcal{G}_V^f + \mathcal{G}_A^f \gamma_5 \right) v_f. \quad (1)$$

With the above results we can now define the *pseudo-observable* quantities that are relevant for the phenomenology of LEP 1/SLC. The *pseudo-variables* are related to measured cross sections and asymmetries by some deconvolution or *unfolding* procedure. The concept of *pseudo-observability* is introduced by saying that the experiments *measure* some primordial (basically cross sections and thereby asymmetries also) quantities, which are then reduced to secondary quantities under some set of specific assumptions. Within these assumptions the secondary quantities, the *pseudo-observables*, also deserve the label of *observability*.

In 1995 the CERN Report [1] on ‘Precision Calculations for the Z resonance’ provided as basic documentation the theoretical basis for upgrading the would-be ’98 Report on Z Physics at LEP 1’.

Although the previous analyses remain quite comprehensive, an update of the discussion of radiative corrections has become necessary for one very good reason: a sizeable amount of theoretical work has appeared following the CERN report of 1995. In particular, a crucial amount of work has been performed in providing higher-order QCD corrections, mixed electroweak and QCD corrections [2], and sub-leading two-loop corrections of $\mathcal{O}(\alpha^2 m_t^2)$ [3].

In ref. [3] the two-loop $\mathcal{O}(\alpha^2 m_t^2)$ corrections are incorporated in the theoretical calculation of M_W and $\sin^2 \theta_{\text{eff}}^l$. More recently the complete calculation of the decay rate of the Z has been made available to us [4]. The only case that is not covered is the one of final b -quarks, because it involves specific $\mathcal{O}(\alpha^2 m_t^2)$ vertex corrections.

Another recent development in the computation of radiative corrections to the hadronic decay of the Z is contained in two papers, which, together, provide complete corrections of $\mathcal{O}(\alpha\alpha_s)$ to $\Gamma(Z \rightarrow q\bar{q})$ with $q = u, d, s, c$ and b . In the first reference of [2] the decay into light quarks is treated. In the second one the remaining diagrams contributing to the decay into bottom quarks are considered and thus the mixed two-loop corrections are complete.

2 Numerical results and comparison

Two of the programs described in the ’95 CERN Report have been constantly updated and we focus, in this note, on a comparison between TOPAZ0 and ZFITTER, with an update of the predictions of

Z -resonance observables within the minimal standard model.

In Table 1 we compare the prediction of TOPAZ0 and ZFITTER for $M_Z = 91.1867$ GeV, $m_t = 175.6$ GeV, $\alpha_s(M_Z^2) = 0.120$ and $M_H = 100$ GeV. The results are from the new versions of the programs, and we have also shown absolute and relative (in per mille) deviations for a set of 25 pseudo-observables. The relative deviation is defined as

$$\delta = 2 \frac{\text{TOPAZ0} - \text{ZFITTER}}{\text{TOPAZ0} + \text{ZFITTER}}. \quad (2)$$

For quantities such as the asymmetries, we report the absolute deviation, which is the only relevant one.

A similar comparison for $M_H = 200$ GeV is shown in Table 2. We observe a deviation in $\sin^2 \theta_{\text{eff}}^l$ of 2.7×10^{-5} (2.3×10^{-5}) for $M_H = 100$ GeV (200 GeV). For the total Z width, the difference between the two programs is of 0.19 MeV (0.08 MeV) for $M_H = 100$ GeV (200 GeV).

In the hadronic Z width the difference is 0.07 MeV at $M_H = 100$ GeV, which roughly corresponds to a variation of $\Delta\alpha_s(M_Z^2) = 0.00013$ in the predictions for $\alpha_s(M_Z^2)$ from the two programs. Variations for the W mass are everywhere below 0.5 MeV.

The level of agreement that is reached is highly satisfactory, especially if we take into account the fact that the implementation of the new correction factors has been performed in a completely independent way, different renormalization schemes and, more important, absolutely different strategies.

It is also interesting to compare the present situation with the differences registered between the two codes at the time of the '95 CERN Report. For this reason we have taken again $M_Z = 91.1888$ GeV, $m_t = 175$ GeV, $\alpha_s(M_Z^2) = 0.125$ and $M_H = 300$ GeV (the '95 input parameter set) and compared some of the predictions. In Tables 3–4 we give the comparison showing, at the same time, the *old-old* and *new-new* deviations and the shifts *old-new*.

It is worth noticing that *new-new* deviations are always less (or much less) than the corresponding *old-old* ones. This fact induces a sizeable reduction of the theoretical uncertainty, achieved after the implementation of the new correction factors. We have also compared our results without the sub-leading terms $\mathcal{O}(\alpha^2 m_t^2)$ and found again the same level of agreement as reached in '95.

In conclusion, we have achieved an important goal: after a substantial upgrading, TOPAZ0 and ZFITTER continue to agree with each other extremely well, in most cases better than they ever have.

An important consequence of this fact is that the central value of the Higgs boson mass in the famous $\chi^2(M_H)$ curve moves down from 115 GeV obtained with old versions to approximately 87 GeV. The difference in predictions between TOPAZ0 and ZFITTER is less than 5 GeV in any of the fits performed so far [5]. Moreover, a substantial reduction is expected of the entire *blue band* that is giving the theoretical uncertainty in the same curve.

3 Acknowledgements

We would like to express special thanks to the TOPAZ0 and ZFITTER teams. Without their contributions the two programs would not be what they are. We both would like to express special thanks to Martin Grünewald, Hans Kühn, Christoph Pauss, and Günter Quast. Finally we acknowledge the

important role played by Giuseppe Degrandi and by Paolo Gambino in helping us with the implementation of the two-loop sub-leading corrections and for sharing with us the result of their work prior to publication.

References

- [1] D. Bardin, W. Hollik and G. Passarino (eds)., Reports of the Working Group on Precision Calculations for the Z-resonance (CERN Report 95-03, Geneva, 1995).
- [2] A. Czarnecki and J.H. Kühn, Phys. Rev. Lett. 77 (1996) 3955 and [hep-ph/9712228](#); R. Harlander, T. Seidensticker and M. Steinhauser, [hep-ph/9712228](#).
- [3] G. Degrandi, S. Fanchiotti and A. Sirlin, *Nucl. Phys.* **B351** (1991) 49;
G. Degrandi and A. Sirlin, *Nucl. Phys.* **B352** (1991) 342;
G. Degrandi, P. Gambino and A. Sirlin, *Phys. Lett.* **B394** (1997) 188;
G. Degrandi, P. Gambino and A. Vicini, *Phys. Lett.* **B383** (1996) 219.
- [4] G. Degrandi and P. Gambino, *in preparation*.
- [5] The LEP Collaborations: ALEPH, DELPHI, L3, OPAL, the LEP Electroweak Working Group and the SLD Heavy Flavour Group. Prepared from Contributions of the LEP and SLD experiments to the 1998 winter conferences, *in preparation*.

	NEW versions			
	TOPAZ0	ZFITTER	Rel. dev. (per mille)	Abs. dev.
$M_H = 100 \text{ GeV}$				
$M_W [\text{GeV}]$	80.3864	80.3860	0.005	0.4 [MeV]
$\Gamma_\nu [\text{MeV}]$	167.235	167.262	-0.16	-0.027 [MeV]
$\Gamma_e [\text{MeV}]$	84.0028	84.0140	-0.13	-0.011 [MeV]
$\Gamma_\mu [\text{MeV}]$	84.0021	84.0133	-0.13	-0.011 [MeV]
$\Gamma_\tau [\text{MeV}]$	83.8110	83.8237	-0.15	-0.013 [MeV]
$\Gamma_u [\text{MeV}]$	300.372	300.387	-0.050	-0.015 [MeV]
$\Gamma_d [\text{MeV}]$	383.161	383.187	-0.068	-0.026 [MeV]
$\Gamma_c [\text{MeV}]$	300.315	300.329	-0.047	-0.014 [MeV]
$\Gamma_b [\text{MeV}]$	376.100	376.082	0.048	0.018 [MeV]
$\Gamma_z [\text{MeV}]$	2496.62	2496.81	-0.076	-0.19 [MeV]
$\Gamma_h [\text{MeV}]$	1743.10	1743.17	-0.040	-0.07 [MeV]
$\Gamma_{\text{inv}} [\text{MeV}]$	501.706	501.787	-0.16	-0.081 [MeV]
$\sin^2 \theta_{\text{eff}}^l$	0.231489	0.231516	-0.12	-0.000027
$\sin^2 \theta_{\text{eff}}^b$	0.232788	0.232902	-0.49	-0.000114
A_{FB}^l	0.0162774	0.0162225	-	0.0000549
A_{FB}^b	0.103232	0.103094	-	0.000138
A_{FB}^c	0.0738276	0.0736708	-	0.0001568
A_{LR}	0.147320	0.147071	-	0.000249
$\sigma_{\text{had}} [\text{nb}]$	41.4717	41.4734	-0.041	-0.0017 [nb]
R_l	20.7505	20.7486	0.092	0.0019
R_b	0.215765	0.215746	0.088	0.000019
R_c	0.172288	0.172289	-0.006	-0.000001
A_{LR}^b	0.934703	0.934638	0.070	0.000065
A_{LR}^c	0.667961	0.667892	0.103	0.000069
s_W^2	0.222855	0.222862	0.031	-0.000007

Table 1: Comparison of TOPAZ0 40i and ZFITTER 510 for $M_H = 100 \text{ GeV}$. Here $M_Z = 91.1867 \text{ GeV}$, $m_t = 175.6 \text{ GeV}$, $\alpha_s(M_Z^2) = 0.120$, $1/\alpha_{\text{em}}(M_Z^2) = 128.896$.

	NEW versions			
	TOPAZ0	ZFITTER	Rel. dev. (per mille)	Abs. dev.
$M_H = 200 \text{ GeV}$				
M_W [GeV]	80.3417	80.3416	0.001	0.1 [MeV]
Γ_ν [MeV]	167.181	167.198	-0.10	-0.017 [MeV]
Γ_e [MeV]	83.9576	83.9635	-0.070	-0.006 [MeV]
Γ_μ [MeV]	83.9569	83.9629	-0.071	-0.006 [MeV]
Γ_τ [MeV]	83.7658	83.7733	-0.090	-0.008 [MeV]
Γ_u [MeV]	300.083	300.080	0.010	0.003 [MeV]
Γ_d [MeV]	382.863	382.866	-0.008	-0.003 [MeV]
Γ_c [MeV]	300.026	300.022	0.013	0.004 [MeV]
Γ_b [MeV]	375.785	375.785	0	0 [MeV]
Γ_Z [MeV]	2494.83	2494.91	-0.032	-0.08 [MeV]
Γ_h [MeV]	1741.61	1741.62	-0.006	-0.01 [MeV]
Γ_{inv} [MeV]	501.542	501.593	-0.10	-0.051 [MeV]
$\sin^2 \theta_{\text{eff}}^l$	0.231849	0.231872	-0.099	-0.000023
$\sin^2 \theta_{\text{eff}}^b$	0.233150	0.233247	-0.42	0.000097
A_{FB}^l	0.0156576	0.0156088	-	0.000488
A_{FB}^b	0.101185	0.101101	-	0.000084
A_{FB}^c	0.0722734	0.0721303	-	0.0001431
A_{LR}	0.144488	0.144263	-	0.000225
σ_{had} [nb]	41.4734	41.4746	-0.029	-0.0012 [nb]
R_l	20.7439	20.7426	0.063	0.0013
R_b	0.215769	0.215768	0.005	0.000001
R_c	0.172269	0.172266	0.017	0.000003
A_{LR}^b	0.934471	0.934416	0.059	0.000055
A_{LR}^c	0.666715	0.666657	0.087	0.000058
s_W^2	0.223719	0.223721	0.009	-0.000002

Table 2: Comparison of TOPAZ0 40i and ZFITTER 510 for $M_H = 200 \text{ GeV}$. Here $M_Z = 91.1867 \text{ GeV}$, $m_t = 175.6 \text{ GeV}$, $\alpha_s(M_Z^2) = 0.120$, $1/\alpha_{\text{em}}(M_Z^2) = 128.896$.

	NEW versus OLD			
	TOPAZ0	ZFITTER	Rel. dev. (per mille)	Abs. dev.
M_W [GeV]				
Old	80.310	80.317	-0.09	-7 [MeV]
New	80.308	80.308	-0.01	-0.1 [MeV]
Rel. shift(per mille)	-0.03	-0.11		
Abs. shift	-2.1 [MeV]	-9.0 [MeV]		
Γ_e [MeV]				
Old	83.931	83.941	-0.12	-0.01 [MeV]
New	83.915	83.920	-0.07	-0.006 [MeV]
Rel. shift(per mille)	-0.19	-0.25		
Abs. shift	-0.016 [MeV]	-0.021 [MeV]		
$\sin^2 \theta_{\text{eff}}^l$				
Old	0.23200	0.23205	-0.22	-5.0×10^{-5}
New	0.23209	0.23211	-0.09	-2.0×10^{-5}
Rel. shift(per mille)	0.39	0.26		
Abs. shift	8.9×10^{-5}	6.1×10^{-5}		
A_{FB}^l				
Old	0.015360	0.015310	-	5.0×10^{-5}
New	0.015249	0.015204	-	4.5×10^{-5}
Abs. shift	-1.1×10^{-4}	-1.1×10^{-4}		
A_{LR}				
Old	0.14327	0.14289	-	3.8×10^{-4}
New	0.14259	0.14238	-	2.1×10^{-4}
Abs. shift	-6.8×10^{-4}	-5.1×10^{-4}		
Γ_Z [MeV]				
Old	2497.4	2497.5	-0.04	-0.1 [MeV]
New	2496.1	2496.2	-0.04	-0.1 [MeV]
Rel. shift(per mille)	-0.52	-0.52		
Abs. shift	-1.29 [MeV]	-1.30 [MeV]		

Table 3: Comparison of TOPAZ0 and ZFITTER. Here $M_Z = 91.1888$ GeV, $m_t = 175$ GeV, $M_H = 300$ GeV, $\alpha_s(M_Z^2) = 0.125$ and $1/\alpha_{\text{em}}(M_Z^2) = 128.896$.

	NEW versus OLD			
	TOPAZ0	ZFITTER	Rel. dev. (per mille)	Abs. dev.
R_l				
Old	20.782	20.781	0.05	0.001
New	20.773	20.773	0	0
Rel. shift(per mille)	-0.41	-0.39		
Abs. shift	-0.0085	-0.008		
R_b				
Old	0.21567	0.21571	-0.19	-4.0×10^{-5}
New	0.21579	0.21580	-0.05	-1.0×10^{-5}
Rel. shift(per mille)	0.57	0.42		
Abs shift	1.2×10^{-4}	9.0×10^{-5}		
R_c				
Old	0.17237	0.17236	0.06	1.0×10^{-5}
New	0.17235	0.17235	0	0
Rel. shift(per mille)	-0.09	-0.06		
Abs. shift	-1.6×10^{-5}	-1.0×10^{-5}		
A_{FB}^b				
Old	0.10033	0.10013	-	2.0×10^{-4}
New	0.099815	0.099767	-	4.8×10^{-5}
Abs. shift	-5.1×10^{-4}	-3.6×10^{-4}		
A_{FB}^c				
Old	0.071590	0.071380	-	2.1×10^{-4}
New	0.071235	0.071100	-	1.3×10^{-4}
Abs. shift	-3.6×10^{-4}	-2.8×10^{-4}		

Table 4: Comparison of TOPAZ0 and ZFITTER. Here $M_Z = 91.1888 \text{ GeV}$, $m_t = 175 \text{ GeV}$, $M_H = 300 \text{ GeV}$, $\alpha_s(M_Z^2) = 0.125$ and $1/\alpha_{\text{em}}(M_Z^2) = 128.896$.